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5D seesaw, flavor structure, and mass textures

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Abstract: In the 5D theory in which only 3 generations of right-handed neutrinos are in the bulk, the neutrino flavor mixings and the mass spectrum can be constructed through the seesaw mechanism. The 5D seesaw is easily calculated just by a replacement of the Majorana mass eigenvalues, M_i, by $2M_* \tan(\mathrm{h})[\pi R M_i]$ (M_*: 5D Planck scale, R: compactification radius). The 5D features appear when the bulk mass, which induces the 4D Majorana mass, is the same as the compactification scale or larger than it. Depending on the type of bulk mass, the seesaw scales of the 3 generations are strongly split (the tan-function case) or degenerate (the tanh-function case). In the split case, the seesaw enhancement is naturally realized. The single right-handed neutrino dominance works in a simple setup, and some specific mass textures, which are just assumptions in the 4D setup, can be naturally obtained in 5 dimensions. The degenerate case is also useful for a suitable neutrino flavor structure.

Keywords: Beyond Standard Model, Neutrino Physics, Compactification and String Models.

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Contents

1. Introduction

Recently, the neutrino oscillation experiments show the detailed flavor structure in the lepton sector [1, 2]. This is one of the most important keys to find physics beyond the standard model (SM). The smallness of the neutrino mass itself is also a clue for new physics. The seesaw mechanism with the heavy Majorana masses of right-handed neutrinos is one of the most reliable candidates for the underlying theory of the SM [3]. The large flavor mixings in the lepton sector can also be realized by the seesaw mechanism. For example, the large Majorana mass hierarchy can induce the large flavor mixings through the seesaw mechanism, even when the Dirac and Majorana neutrino mass matrices have small flavor mixings [4]. This is the typical example of a so-called seesaw enhancement [5]. The single right-handed neutrino dominance (SRND) [6] should be classified as the seesaw enhancement, in which the lightest Majorana mass is the dominant contribution of the seesaw and induces the large flavor mixing with the Dirac mass matrix structure. There are a lot of other attempts to obtain the suitable neutrino flavor structure through the seesaw mechanism from the viewpoint of mass textures, flavor symmetries, and so on.

On the other hand, the large extra dimensional theory has been proposed to solve the gauge hierarchy problem [7]. This theory explains the smallness of Dirac neutrino mass due to the existence of right-handed neutrinos in the bulk [8]. It suggests new possibilities for the neutrino physics [9]. In the framework of the extra dimensional theory, we can also consider the seesaw mechanism by introducing a bulk mass for the right-handed neutrinos which induces the 4D Majorana mass [10]–[12].

In this paper, we adopt the 5D setup[1] in which the 3 generations of right-handed neutrinos are in the bulk with bulk mass. We do not specify the size of the compactification scale so that our discussions are applicable to both large and small extra dimensional

[1]The neutrino flavor structure has been discussed in the similar setup by introducing the S_3 flavor symmetry and its breaking through a boundary twist [13].

theories. We show a simple calculation method of the 5D seesaw, which just replaces the 4D Majorana mass eigenvalues, M_i, by $2M_* \tan(\mathrm{h})[\pi R M_i]$ (M_*: 5D Planck scale, R: compactification radius). The neutrino flavor mixings and the mass spectrum can be drastically changed by the 5D seesaw. The 5D features appear when the Majorana mass is the same as the compactification scale or larger than it. Depending on the type of bulk mass, the seesaw scales of the 3 generations are strongly split (the tan-function case) or degenerate (the tanh-function case). In the split case, the seesaw enhancement is realized naturally. We show that the SRND works in a simple setup in 5 dimensions. This split also makes the tiny symmetry breaking induce the suitable flavor structure in the $L_\mu - L_\tau$ symmetry [14, 15]. We also show that some specific mass textures, which the 4D theory just assumes, can be naturally obtained in the 5D seesaw. The degenerate case can also realize the suitable flavor structure.

2. Setup and 5D seesaw

We take the 5D setup, where the 5th dimension coordinate (y) is compactified on S^1/Z_2. Only right-handed neutrinos, which are the SM gauge singlets, are spread in the 5D bulk, while the other quarks and leptons are localized on the 4D brane. Since the 5D theory is a vector-like theory, we must introduce the chiral partners, N_i^c, of the right-handed neutrinos, N_i, where N_i^c and N_i are the two component Weyl spinors of the ith generation ($i = 1, 2, 3$). Under Z_2 parity, $y \to -y$, $\mathcal{N} \equiv (N, N^c)^T$ transforms as $\mathcal{N}(x^\mu, -y) = \gamma^5 \mathcal{N}(x^\mu, y)$. Then the mode expansion is given by

$$\mathcal{N}_i(x^\mu, y) \;=\; \frac{1}{\sqrt{\pi R}} \begin{pmatrix} \frac{1}{\sqrt{2}} N_i^{(0)}(x^\mu) + \sum\limits_{n=1}^{\infty} \cos(\frac{ny}{R})\, N_i^{(n)}(x^\mu) \\ \sum\limits_{n=1}^{\infty} \sin(\frac{ny}{R})\, N_i^{c(n)}(x^\mu) \end{pmatrix}, \tag{2.1}$$

where the factor $1/\sqrt{2}$ of the zero-mode is needed for the canonical kinetic term in the 4D effective Lagrangian. Hence, only right-handed neutrinos, N_i's, have the zero-modes and survive below the compactification scale, R^{-1}.

The general 5D bulk mass terms which induce the 4D Majorana masses are given by [11]

$$\mathcal{L}_{5\mathrm{DM}} \;=\; -\frac{1}{2}\left(M_{ij}^S\, \overline{\mathcal{N}_i^c}\mathcal{N}_j + M_{ij}^V\, \overline{\mathcal{N}_i^c}\gamma_5\mathcal{N}_j + \mathrm{h.c.}\right). \tag{2.2}$$

Here, M^S and M^V are symmetric matrices for the generation index, i, j. They are invariant under the Z_2 parity but break the lepton number. M^V is not 5D Lorentz invariant but derivable from the spontaneous symmetry breaking of the 5D Lorentz invariance [11].

By integrating out the 5th dimension, the 4D effective Lagrangian is obtained, in which the Majorana mass terms are given by

$$\mathcal{L}_{4\mathrm{DM}} \;=\; -\frac{1}{2}\left(\sum_{n=0}^{\infty} M_{ij}^A {N_i}^{(n)T} {N_j}^{(n)} + \sum_{n=1}^{\infty} M_{ij}^{B*} \overline{N_i^c}^{(n)T} \overline{N_j^c}^{(n)}\right) + \mathrm{h.c.}, \tag{2.3}$$

where $M^A \equiv M^S + M^V$ and $M^{B*} \equiv -M^{S*} + M^{V*}$. We omit spinor indices here. Kaluza-

Klein (KK) masses are given by

$$\mathcal{L}_{\text{KK}} \;=\; -\sum_{n=1}^{\infty} \frac{n}{R} \,(\overline{N_i^c}^{(n)} N_i{}^{(n)} + \overline{N_i}^{(n)} N_i^{c(n)}). \tag{2.4}$$

The Dirac mass terms between the bulk right-handed neutrinos, N_i and N_i^c, and the brane-localized lepton doublets, L_i, are given by

$$\mathcal{L}_{\text{4Dm}} = -\frac{1}{\sqrt{M_*}}(\overline{L_i}\, m_{ij}\, N_j \;+\; \overline{L_i}\, m_{ij}^c\, N_j^c\,)\delta(y) + \text{h.c.}, \tag{2.5}$$

where $m_{ij} = y_{ij}\langle H\rangle$ and $m_{ij}^c = y_{ij}^c\langle H\rangle$. $\langle H\rangle$ is the vacuum expectation value (VEV) of the SM Higgs doublet H which is localized on the 4D brane, and y_{ij} and y_{ij}^c are Yukawa couplings. In our setup, $m_{ij}^c = 0$ due to the Z_2 parity (mode expansion in eq. (2.1)). Then the 4D neutrino mass matrix is given by

$$\mathcal{L}_{m_\nu} \;=\; -\frac{1}{2}\,\nu_i^T\,(\mathcal{M}_\nu)_{ij}\,\nu_j \;+\text{h.c.},$$

$$\mathcal{M}_\nu \;=\; \left(\begin{array}{cccc|cccc}
0 & 0 & 0 & \cdots & \frac{m_{ij}}{\sqrt{2\pi R M_*}} & \frac{m_{ij}}{\sqrt{\pi R M_*}} & \frac{m_{ij}}{\sqrt{\pi R M_*}} & \cdots \\
 & M_{ij}^{B*} & & & & \frac{1}{R} & & \\
 & & M_{ij}^{B*} & & & & \frac{2}{R} & \\
\vdots & & & \ddots & & & & \ddots \\
\hline
\frac{(m^T)_{ij}}{\sqrt{2\pi R M_*}} & & & & M_{ij}^A & & & \\
\frac{(m^T)_{ij}}{\sqrt{\pi R M_*}} & \frac{1}{R} & & & & M_{ij}^A & & \\
\frac{(m^T)_{ij}}{\sqrt{\pi R M_*}} & & \frac{2}{R} & & & & M_{ij}^A & \\
\vdots & & & \ddots & & & & \ddots
\end{array}\right), \tag{2.6}$$

in the basis of $\nu_i^T \equiv (\overline{L_i}, \overline{N_i^c}^{(1)}, \overline{N_i^c}^{(2)}, \cdots | N_i^{(0)}, N_i^{(1)}, N_i^{(2)}, \cdots)$. Let us adopt M^V for the bulk mass, and pick up the sub-matrix of n-mode from eq. (2.6). Since the KK mass, n/R, is proportional to the unit matrix in the flavor space, all n-modes of the right-handed neutrinos are diagonalized simultaneously in the flavor space as

$$\begin{pmatrix} U_R^\dagger & \\ & U_R^T \end{pmatrix} \begin{pmatrix} M_{ij}^{V*} & \frac{n}{R} \\ \frac{n}{R} & M_{ij}^V \end{pmatrix} \begin{pmatrix} U_R^* & \\ & U_R \end{pmatrix}, \tag{2.7}$$

where U_R is the unitary matrix which diagonalizes M_{ij}^V as $U_R^T M_{ij}^V U_R = M_d$. The three eigenvalues of M_d can be written as $M_i = |M_i| e^{i\alpha_i}$ $(i = 1, 2, 3)$[2]. In this basis, the Dirac mass matrix is given by $(0 \;\; mU_R)$. We denote $m_R \equiv mU_R$ hereafter. Since the inverse mass matrix of eq. (2.7) is given by

$$\frac{1}{|M_i|^2 - (\frac{n}{R})^2} \begin{pmatrix} M_i & -\frac{n}{R} \\ -\frac{n}{R} & M_i^* \end{pmatrix}, \tag{2.8}$$

[2]We take the basis where the phase α_i remains in M_i in eqs. (2.8) and (2.9). It is also possible to absorb this phase by using the unitary matrix $U_R P$, where $P_{ii} = e^{-i\alpha_i/2}$. In this basis M_i is real and positive in eq. (2.7), and then the Dirac mass matrix becomes $m_R P$ which induces the phase $e^{-i\alpha_k}$ in eq. (2.9).

the summation of the infinite numbers of "seesaw" is calculated as

$$m^{\nu}_{ij} = -\frac{(m_R)_{ik}}{\pi R M_*}\left[\frac{1}{2M_k} + \sum_{n=1}^{\infty}\frac{M_k^*}{|M_k|^2 - (\frac{n}{R})^2}\right](m_R^T)_{kj}$$
$$= -(m_R)_{ik}\,\frac{e^{-i\alpha_k}}{2M_*\tan[\pi R|M_k|]}\,(m_R^T)_{kj}, \tag{2.9}$$

when the magnitude of $\min[|M_i| \pm \frac{n}{R}]$ is much larger than the Dirac mass scale. The case of $|M_i| \ll R^{-1}$ reproduces the ordinal 4D seesaw formula with the volume suppression factor, $2\pi R M_*$.

To give a short summary, in order to get the 5D seesaw mass matrix, we just make the replacement

$$M_i \to 2M_* \tan[\pi R|M_i|]\, e^{i\alpha_i} \tag{2.10}$$

in the Majorana mass diagonal basis. By use of eq. (2.10), we can immediately achieve the 5D seesaw calculations. When we adopt M^S for a bulk mass in eq. (2.2), the replacement in eq. (2.10) is modified as

$$M_i \to 2M_* \tanh[\pi R|M_i^S|]\, e^{i\beta_i}, \tag{2.11}$$

where $M_i^S (= |M_i^S| e^{i\beta_i})$ is the mass eigenvalue of M_{ij}^S[3]. Anyhow, the tan(h)-function is a feature in the 5D seesaw, which has the significant effects when $|M_i|, |M_i^S| \geq \mathcal{O}(R^{-1})$. The typical case is $|M_i| = |\frac{2n+1}{2R}|$ (n: integer) where the light neutrino becomes massless. It is because the infinite times seesaw becomes $-\frac{1}{2\pi R M_*} \times ([\frac{1}{2R}]^{-1} - [\frac{1}{2R}]^{-1} + [\frac{3}{2R}]^{-1} - [\frac{3}{2R}]^{-1} + \cdots) \to 0$. Notice that n should satisfy $|n + \frac{1}{2}| \ll RM_*$ to fulfill $|M_i| \ll M_*$.

When $|M_i| \geq \mathcal{O}(R^{-1})$, the largest contribution to the seesaw is not from $|M_i|$ but from $\min[|M_i| \pm \frac{n}{R}] (\leq \frac{1}{2R})$, which is the origin of the periodic tan-function [10]. On the other hand, the 5D Planck scale plays a role of seesaw scale when $|M_i^S| \geq \mathcal{O}(R^{-1})$. Therefore, in the case of $|M_i|, |M_i^S| \geq \mathcal{O}(R^{-1})$, the hierarchy of the seesaw scales can be different from the original mass hierarchies of M_i, M_i^S. For example, suppose $M_1^{(S)} < M_2^{(S)} < M_3^{(S)}$ for the original Majorana mass hierarchy, then, the hierarchy of the 5D seesaw scales becomes acceleratingly split, or even arbitrary from the periodicity, (degenerate) due to the tan(h)-function.

If we adopt both M^V and M^S for the bulk mass simultaneously, the seesaw calculation becomes complicated. However, if they are diagonalized by the same unitary matrix U_R, we can obtain the simple 5D seesaw formula. Taking the Majorana mass eigenvalues to be real for simplicity[4], the formula becomes

$$m^{\nu}_{ij} = -(m_R)_{ik}\,\frac{M_k - M_k^S}{2M_* M_c \tan[\pi R M_c]}\,(m_R^T)_{kj} \qquad (|M_k| > |M_k^S|), \tag{2.12}$$
$$= -(m_R)_{ik}\,\frac{M_k^S - M_k}{2M_* M_c' \tanh[\pi R M_c']}\,(m_R^T)_{kj} \qquad (|M_k| < |M_k^S|), \tag{2.13}$$

[3]Equation (2.10) ((2.11)) is consistent with the results in ref. [10] ([11]), where M^V (M^S) is taken to be real.

[4]If they are complex, the formula is given by $-(m_R)_{ik}\frac{(M_k^* - M_k^{S*})}{2M_* M_k' \tan[\pi R M_k']}(m_R^T)_{kj}$ where $M_k' \equiv ((M_k^* - M_k^{S*})(M_k + M_k^S))^{1/2}$. We must take the same phase for two M_k's (square root of the complex number) in the denominator.

where $M_c \equiv \sqrt{M_k^2 - M_k^{S2}}$ and $M_c' \equiv \sqrt{M_k^{S2} - M_k^2}$. When $|M_k| > |M_k^S|$ ($|M_k| < |M_k^S|$), the denominator has a tan(h)-function. Thus, it is possible that different generations have different seesaw scales characterized by the tan- or tanh-function in general. We take either M^V or M^S for a bulk mass of the right-handed neutrinos in the following discussions, but we can always replace M^V (M^S) by two real masses of M^V and M^S with eq. (2.12) (eq. (2.13)).

3. Flavor structure from 5D seesaw

In this section we show some 5D seesaw examples which are just assumptions or impossible in the 4D setup. We show useful examples of the tan(h)-function for the neutrino flavor structure.

3.1 Mass textures

At first, we show two examples of neutrino mass textures when one Majorana mass eigenvalue satisfies $|M_i| = |\frac{2n+1}{2R}|$[5] ($|n + \frac{1}{2}| \ll RM_*$). This case makes the ith column of the Dirac mass matrix, m_{ij}, vanish from the seesaw calculation.

For a specific mass texture, there should exist the flavor symmetry as the symmetry of the underlying theory of the SM. Here, let us consider the S_3 flavor symmetry [16] [17]. The setup is designed in a way that both right- and left-handed neutrinos are $\mathbf{3}(= \mathbf{2} + \mathbf{1_S})$ representations of S_3, in which the 1st and 2nd generations are doublets ($\mathbf{2}$) and the 3rd generation is the singlet ($\mathbf{1_S}$) in the complex basis. We consider two kinds of $\mathbf{3}(= \mathbf{2} + \mathbf{1_S})$ S_3-Higgs fields, ϕ^D and ϕ^N, in the bulk, where we denote the doublet as $\phi_{1,2}^{D,N}$ and the singlet as $\phi_S^{D,N}$. ϕ^N possesses the lepton number while ϕ^D does not. We set ϕ^D as a $SU(2)_L$ singlet to avoid large lepton flavor violations (and also to use ϕ^* in eq. (3.1)[6]). Then, the general Dirac and Majorana mass matrices become [17]

$$m = \begin{pmatrix} x\phi_S^D + x'\phi_S^{D*} & z\phi_1^D + z'\phi_2^{D*} & w\phi_2^D + \xi'\phi_1^{D*} \\ z\phi_2^D + z'\phi_1^{D*} & x\phi_S^D + x'\phi_S^{D*} & w\phi_1^D + \xi'\phi_2^{D*} \\ \xi\phi_1^D + w'\phi_2^{D*} & \xi\phi_2^D + w'\phi_1^{D*} & y\phi_S^D + y'\phi_S^{D*} \end{pmatrix}, \tag{3.1}$$

$$M = \begin{pmatrix} \alpha\phi_1^N & \gamma\phi_S^N & \eta\phi_2^N \\ \gamma\phi_S^N & \alpha\phi_2^N & \eta\phi_1^N \\ \eta\phi_2^N & \eta\phi_1^N & \rho\phi_S^N \end{pmatrix}, \tag{3.2}$$

respectively. Where ϕ^D and ϕ^N stand for their VEVs of zero-modes, which could be determined by the S_3-Higgs potential analysis in principle. $x, x', \cdots, \eta, \rho$ are independent Yukawa couplings, and those of eq. (3.1) ((3.2)) are proportional to $\frac{\langle H \rangle}{2\pi RM_*^2}$ ($\frac{1}{\sqrt{2\pi RM_*}}$). For the Dirac mass matrix, L_i (N_i) is taken from the left- (right-) hand side as in eq. (2.5).

The first example is the neutrino mass texture which induces the inverted hierarchy (IH) mass spectrum [18]. We introduce only S_3 singlet Higgs, ϕ_S^N, for the Majorana mass. As for the Dirac sector, we introduce both, ϕ_S^D and $\phi_{1,2}^D$, and assume $z = w' = 0$ as well as

[5] The Majorana mass is not necessarily exact $|M_i| = |\frac{2n+1}{2R}|$. Close enough to this value the following mechanisms work well.

[6] When ϕ has some other quantum charges than S_3, ϕ^* is removed from the mass matrix elements [17].

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a vanishing VEV, $\phi_1^D = 0$. Then the Dirac and Majorana mass matrices are given by

$$m = \begin{pmatrix} A & E & D \\ 0 & B & F \\ 0 & C & G \end{pmatrix}, \quad M = \begin{pmatrix} & X & \\ X & & \\ & & Y \end{pmatrix}, \tag{3.3}$$

respectively. We denote $X = |X|e^{i\theta_X}$ and $Y = |Y|e^{i\theta_Y}$. In the diagonal basis of Majorana mass matrix, the matrices are rewritten as

$$m_R = \frac{1}{\sqrt{2}} \begin{pmatrix} A-E & A+E & \sqrt{2}D \\ -B & B & \sqrt{2}F \\ -C & C & \sqrt{2}G \end{pmatrix}, \quad M_d = \begin{pmatrix} -X & & \\ & X & \\ & & Y \end{pmatrix}, \tag{3.4}$$

which suggest for the 5D seesaw,

$$\begin{aligned} m_\nu^{5\mathrm{D}} &= -m_R\ (M_d)^{-1}\ m_R^T, \\ &= -\frac{1}{4M_*}\left[\delta_1 \begin{pmatrix} A-E \\ -B \\ -C \end{pmatrix} (A-E\ \ -B\ \ -C) + \delta_2 \begin{pmatrix} A+E \\ B \\ C \end{pmatrix} (A+E\ B\ G)\right. \\ &\qquad\qquad \left. + \delta_3 \begin{pmatrix} \sqrt{2}D \\ \sqrt{2}F \\ \sqrt{2}G \end{pmatrix} (\sqrt{2}D\ \sqrt{2}F\ \sqrt{2}G)\right], \end{aligned} \tag{3.5}$$

where $\delta_1 = -\delta_2 = -\cot[\pi R|X|]e^{-i\theta_X}$ and $\delta_3 = \cot[\pi R|Y|]e^{-i\theta_Y}$. The ith term is the seesaw contribution from M_i $(i = 1, 2, 3)$. In the case of $|Y| = |\frac{2n+1}{2R}|$[7], the 3rd generation right-handed neutrino becomes heavy and decouples, and hence the 3rd term of eq. (3.5) vanishes. Then, the neutrino mass texture becomes

$$m_\nu^{5\mathrm{D}} \simeq \frac{\delta_1}{2M_*} \begin{pmatrix} 2AE & AB & AC \\ AB & 0 & 0 \\ AC & 0 & 0 \end{pmatrix}. \tag{3.6}$$

The suitable mass texture for the IH mass spectrum is obtained when $|E| \ll |A|, |B|, |C|$ and $|B| \simeq |C|$.

The second example is the neutrino mass texture with only two right-handed neutrinos [19]. The introduction of only two right-handed neutrinos seems artificial. However, in the 5D setup, this texture can be naturally realized as follows:

Introducing only an S_3 doublet Higgs, $\phi_{1,2}^D$, the diagonal elements of the Dirac mass matrix vanish. As for the Majorana mass matrix, we introduce both ϕ_S^N and $\phi_{1,2}^N$, and assume $\gamma = \eta = 0$. Then the Dirac and Majorana mass matrices are given by

$$m = \begin{pmatrix} 0 & A & B \\ C & 0 & D \\ E & F & 0 \end{pmatrix}, \quad M = \begin{pmatrix} X & & \\ & X' & \\ & & Y \end{pmatrix}, \tag{3.7}$$

[7] Using $\langle\phi_S^N\rangle$, this condition is rewritten as $|\rho\langle\phi_S^N\rangle| = \sqrt{\frac{\pi(2n+1)^2}{2}R^{-1}M_*}$.

respectively, where we denote $X = |X|e^{i\theta_X}$, $X' = |X'|e^{i\theta'_X}$, and $Y = |Y|e^{i\theta_Y}$. Then, the 5D seesaw is given by

$$m_\nu^{\rm 5D}{=}-\frac{1}{2M_*}\left[\delta'_1\begin{pmatrix}0\\C\\E\end{pmatrix}(0\;C\;E)+\delta'_2\begin{pmatrix}A\\0\\F\end{pmatrix}(A\;0\;F)+\delta'_3\begin{pmatrix}B\\D\\0\end{pmatrix}(B\;D\;0)\right], \tag{3.8}$$

where $\delta'_1 = \cot[\pi R|X|]e^{-i\theta_X}$, $\delta'_2 = \cot[\pi R|X'|]e^{-i\theta'_X}$, and $\delta'_3 = \cot[\pi R|Y|]e^{-i\theta_Y}$. In the case of $|X'| = |\frac{2n+1}{2R}|$, the 2nd generation right-handed neutrino decouples and the 2nd term in eq. (3.8) vanishes. Then the neutrino mass texture becomes

$$m_\nu^{\rm 5D} = -\frac{\delta'_1}{2M_*}\begin{pmatrix} rB^2 & rBD & 0\\ rBD & rD^2{+}C^2 & CE\\ 0 & CE & E^2\end{pmatrix}, \tag{3.9}$$

where $r \equiv \frac{\delta'_3}{\delta'_1}$. For $|B| \sim |D|$ and $|C| \simeq |E|$ with $|r| \sim 0.1 \times \frac{|C^2|}{|D^2|}$, bi-large mixing with the normal hierarchy (NH) mass spectrum is realized [19].

3.2 SRND in $L_\mu - L_\tau$ symmetry

When $|M_i| \sim |\frac{2n+1}{2R}|$, the tan-function strongly splits the effective seesaw scales, which is useful for the SRND. As an example of the SRND, let us discuss the neutrino mass texture in the $L_\mu - L_\tau$ symmetry. In the 4D case, the seesaw mechanism needs a large symmetry breaking parameter in the Majorana mass matrix to realize the NH mass spectrum [15]. On the other hand, the 5D seesaw can enhance a small symmetry breaking and realize the NH mass spectrum with the suitable flavor mixings as follows:

At first, let us show the 2-3 generation sub-matrix. Under the $L_\mu - L_\tau$ symmetry, the Dirac and Majorana mass matrices are given by [15]

$$m = \begin{pmatrix} A & 0\\ 0 & B\end{pmatrix}, \quad M = \begin{pmatrix}\varepsilon & X\\ X & \varepsilon\end{pmatrix}, \tag{3.10}$$

respectively, where A, B, X are $L_\mu - L_\tau$ symmetric mass parameters. We take X to be real and positive for simplicity. $\varepsilon\ (> 0)$ is the symmetry breaking parameter which should be smaller than the symmetric one X. In the basis where the Majorana mass matrix is diagonal the seesaw formula is given by

$$m_\nu = -\frac{1}{2}\begin{pmatrix} A & A\\ -B & B\end{pmatrix}\begin{pmatrix}\frac{1}{\varepsilon-X} & 0\\ 0 & \frac{1}{\varepsilon+X}\end{pmatrix}\begin{pmatrix} A & -B\\ A & B\end{pmatrix}. \tag{3.11}$$

For the realization of a NH mass spectrum, large symmetry breaking, $\varepsilon \sim X$, is required [15].

The 5D seesaw can avoid this unnatural large symmetry breaking. By using eq. (2.10), we can obtain the 5D seesaw directly from eq. (3.11) as

$$m_\nu^{\rm 5D} = -\frac{1}{4M_*}\begin{pmatrix} A & A\\ -B & B\end{pmatrix}\begin{pmatrix}\frac{1}{\tan[\pi R(\varepsilon-X)]} & 0\\ 0 & \frac{1}{\tan[\pi R(\varepsilon+X)]}\end{pmatrix}\begin{pmatrix} A & -B\\ A & B\end{pmatrix}. \tag{3.12}$$

When $X \simeq \frac{1}{2R}$, even small ε can produce a Majorana mass hierarchy that is large enough for the SRND to work. In the case of $(\varepsilon + X) = \frac{1}{2R}$, $|\varepsilon - X|$ is also around $\frac{1}{2R}$ but still effective for the seesaw. Then eq. (3.12) becomes

$$m_\nu^{5\mathrm{D}} \simeq -\frac{1}{4M_* \tan[\pi R(\varepsilon - X)]} \begin{pmatrix} A \\ -B \end{pmatrix} (A \ -B), \tag{3.13}$$

which suggests the NH spectrum. Maximal mixing will be obtained in the case of $|A| \simeq |B|$, even if $\varepsilon \ll X$.

The suitable 3-generation flavor structure is obtained by introducing a symmetry breaking parameter, C', in the Dirac mass matrix as

$$m = \begin{pmatrix} C & & \\ C' & A & \\ & & B \end{pmatrix}, \quad M = \begin{pmatrix} Y & & \\ & \varepsilon & X \\ & X & \varepsilon \end{pmatrix}. \tag{3.14}$$

Taking Y to be real and positive as $Y < X$, the 5D seesaw induces the neutrino mass matrix

$$m_\nu^{5\mathrm{D}} \simeq -\frac{1}{4M_*} \begin{pmatrix} \frac{2C^2}{\pi RY} & \frac{2CC'}{\pi RY} & 0 \\ \frac{2CC'}{\pi RY} & \frac{A^2}{\tan[\pi R(\varepsilon - X)]} + \frac{2C'^2}{\pi RY} & -\frac{AB}{\tan[\pi R(\varepsilon - X)]} \\ 0 & -\frac{AB}{\tan[\pi R(\varepsilon - X)]} & \frac{B^2}{\tan[\pi R(\varepsilon - X)]} \end{pmatrix}. \tag{3.15}$$

In the case of $|C| \sim |C'|$ and $|A| \simeq |B|$ with $\frac{2|C'^2|}{\pi RY} \sim 0.1 \times \frac{|A^2|}{\tan[\pi R(X-\varepsilon)]}$, the suitable bi-large mixing is obtained. Although the partial degeneracy is needed in the Dirac mass matrix we can realize bi-large mixing with the NH mass spectrum by only two non-large symmetry breaking parameters.

3.3 SRND in $U(1)_F$ flavor symmetry

In the previous sub-section, we have shown one example of the SRND when the bulk mass was M^V. This sub-section shows some more examples with the bulk mass M^V or M^S by introducing a $U(1)_F$ flavor symmetry. The following discussion does not change if we take a discrete subgroup of the $U(1)_F$ such as Z_N with enough large N and the same charge assignments. We will show that the SRND can work in a much simpler setup in 5 dimensions.

We introduce only one scalar bulk field, ϕ, with $U(1)_F$ charge, -1. The zero-mode of ϕ takes a real and positive constant VEV as

$$\frac{\langle \phi \rangle}{\sqrt{2\pi R M_*^3}} \equiv \varepsilon \ (< 1). \tag{3.16}$$

Taking the charge assignment of the lepton sector as

$$L_1 : 1, \quad L_2 : 0, \quad L_3 : 0, \quad N_1 : 2, \quad N_2 : 1, \quad N_3 : 0, \tag{3.17}$$

the Dirac and Majorana mass matrices are given by

$$m \simeq \begin{pmatrix} \varepsilon^3 & \varepsilon^2 & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix} m_0, \quad M \simeq \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix} M_0, \tag{3.18}$$

respectively. We take both m_0 and M_0 to be real and positive for simplicity. They represent only the order, and each element has $\mathcal{O}(1)$ coefficient. Thus, the ranks of the mass matrices are both 3 in general. In the diagonal basis of the Majorana mass matrix, eq. (3.18) becomes

$$m_R \simeq \begin{pmatrix} \varepsilon^3 & \varepsilon^2 & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix} m_0, \quad M_d \simeq \begin{pmatrix} \varepsilon^4 & & \\ & \varepsilon^2 & \\ & & 1 \end{pmatrix} M_0. \tag{3.19}$$

Then, the seesaw formula is given by

$$m_\nu \simeq -\frac{m_0^2}{M_0}\left[\frac{1}{\varepsilon^4}\begin{pmatrix} \varepsilon^3 \\ \varepsilon^2 \\ \varepsilon^2 \end{pmatrix}(\varepsilon^3\ \varepsilon^2\ \varepsilon^2) + \frac{1}{\varepsilon^2}\begin{pmatrix} \varepsilon^2 \\ \varepsilon \\ \varepsilon \end{pmatrix}(\varepsilon^2\ \varepsilon\ \varepsilon) + \begin{pmatrix} \varepsilon \\ 1 \\ 1 \end{pmatrix}(\varepsilon\ 1\ 1)\right]. \tag{3.20}$$

This mass matrix can potentially induce bi-large mixing with a NH mass spectrum. However, the mass spectrum is naively $\varepsilon^2 : 1 : 1$ in units of $\frac{m_0^2}{M_0}$, and hence, in order to realize the NH spectrum, the determinant of the 2-3 sub-matrix must be reduced to $\mathcal{O}(\varepsilon)$ by tuning of $\mathcal{O}(1)$ coefficients. The SRND could avoid this tuning, but it does not work in this setup because the three terms in eq. (3.20) are all of the same order.

Now let us examine the 5D case. Thanks to the 5D feature, the tan(h)-function can change the orders of three terms in eq. (3.20). At first, we adopt M^S for the bulk mass. In this case the 5D seesaw suggests

$$m_\nu^{5\mathrm{D}} \simeq -\frac{m_0^2}{2M_* \tanh[\pi R M_0]} \times \left[\delta_1^S \begin{pmatrix} s\varepsilon \\ t \\ u \end{pmatrix}(s\varepsilon\quad t\quad u) + \delta_2^S \begin{pmatrix} s'\varepsilon \\ t' \\ u' \end{pmatrix}(s'\varepsilon\quad t'\quad u') + \begin{pmatrix} a\varepsilon \\ b \\ c \end{pmatrix}(a\varepsilon\quad b\quad c)\right], \tag{3.21}$$

where $\delta_1^S = \frac{\varepsilon^4 \tanh[\pi R M_0]}{\tanh[\pi R \varepsilon^4 M_0]}$, $\delta_2^S = \frac{\varepsilon^2 \tanh[\pi R M_0]}{\tanh[\pi R \varepsilon^2 M_0]}$, and $s, t, \cdots b, c$ are $\mathcal{O}(1)$ coefficients in which the coefficients of M_i^S are absorbed. $\delta_{1,2}^S \simeq 1$ when $\varepsilon^2 M_0 \ll R^{-1}$, while $\delta_1^S \simeq \varepsilon^4$ and $\delta_2^S \simeq \varepsilon^2$ when $\varepsilon^4 M_0 \gg R^{-1}$. The SRND structure is obtained when $\delta_{1,2}^S \sim 0.1$, in which the reduction of the 2-3 sub-matrix determinant is automatic and the mass spectrum becomes $\delta_{1,2}^S \varepsilon^2 : \delta_{1,2}^S : 1$ in units of $\frac{m_0^2}{2M_* \tanh[\pi R M_0]}$.

It should be noticed that the dominant contribution to the light neutrino mass matrix is coming from the heaviest Majorana mass, M_3^S, which is completely different from the ordinal SRND in the 4D setup. This is new type of the SRND realized by the 5D feature, the tanh-function. Precisely speaking, this is also possible even in the 4D setup in eq. (3.20), if $\frac{|\varepsilon^2 M_3|}{|M_2|} \sim \frac{|\varepsilon^4 M_3|}{|M_1|} \sim 0.1$ is realized accidentally by the diagonalization of Majorana mass matrix M (eq. (3.19)). We stress that the 5D setup can naturally realize the SRND structure without tuning of $\mathcal{O}(1)$ coefficients in the Majorana mass matrix.

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For the suitable mixing angles, we should choose $\mathcal{O}(1)$ coefficients [20]. For the 2-3 maximal mixing, the condition $|b| \simeq |c| (\sim 1)$ is needed. Remembering that bi-large mixing with the NH mass spectrum needs the following four types of sign assignments

$$\begin{pmatrix} \varepsilon^2 & \mp\varepsilon & \pm\varepsilon \\ \mp\varepsilon & 1 & 1 \\ \pm\varepsilon & 1 & 1 \end{pmatrix}, \qquad \begin{pmatrix} \varepsilon^2 & \pm\varepsilon & \pm\varepsilon \\ \pm\varepsilon & 1 & -1 \\ \pm\varepsilon & -1 & 1 \end{pmatrix}, \tag{3.22}$$

we can show that the 3rd term in eq. (3.21) alone cannot realize the bi-large mixing. Corrections from the 1st and 2nd terms in eq. (3.21) are needed [6] for the suitable signs of eq. (3.22). For example, to flip the sign of the (2,1), (1,2) elements, the relation $|ab| < |\delta_1^S st + \delta_2^S s't'|$ should be satisfied, which is impossible when $\delta_{1,2}^S \ll 1$. However, the experimental data implies that $\delta_{1,2}^S$ is not so small, $\delta_{1,2}^S \sim \sqrt{\Delta m_\odot^2/\Delta m_{\rm A}^2}$, in the NH mass spectrum. Hence, the above condition can be satisfied even by $\mathcal{O}(1)$ coefficients. We should take relatively small (large) values of a (s, t, s', t') as the $\mathcal{O}(1)$ coefficients [20]. For example, $a = u' = 0.5, s = s' = 4, b = c = -t = u = -t' = 1$ with $\varepsilon = 0.2$ and $M_0 = 3.3R^{-1}$ ($\delta_{1,2}^S \simeq 0.1$) induce $\theta_{12} = 33°$, $\theta_{23} = 46°$, $\theta_{13} = 3.3°$, and $\sqrt{\Delta m_\odot^2/\Delta m_{\rm A}^2} = 0.22$.

Next, let us adopt M^V for the bulk mass. Remembering that the seesaw scales are strongly split when $|M_i| \sim |\frac{2n+1}{2R}|$, we assume

$$|M_1| \ll \frac{1}{2R}, \quad |M_2| \simeq \frac{1}{2R}, \quad |M_3| \simeq \left|\frac{2n+1}{2R}\right|. \tag{3.23}$$

The neutrino mass matrix is given by

$$m_\nu^{\rm 5D} \simeq -\frac{m_0^2}{2\pi R M_* M_0}\left[\begin{pmatrix}\varepsilon\\1\\1\end{pmatrix}(\varepsilon \;\; 1 \;\; 1) + \delta\begin{pmatrix}\varepsilon\\1\\1\end{pmatrix}(\varepsilon \;\; 1 \;\; 1) + \hat{\delta}\begin{pmatrix}\varepsilon\\1\\1\end{pmatrix}(\varepsilon \;\; 1 \;\; 1)\right], \tag{3.24}$$

where $\delta = \frac{\pi R\varepsilon^2 M_0}{\tan[\pi R|M_2|]}$ and $\hat{\delta} = \frac{\pi R M_0}{\tan[\pi R|M_3|]}$. Thanks to the 5D feature, the tan-function, the SRND structure can be obtained with $\delta \sim \hat{\delta} \sim 0.1$. This is the similar structure to eq. (3.21), in which the reduction of the 2-3 sub-matrix determinant is automatically achieved with the mass spectrum $\delta\varepsilon^2 : \delta : 1$ in units of $\frac{m_0^2}{2\pi R M_* M_0}$. $\mathcal{O}(1)$ coefficients can induce the suitable mixing angles [20].

The above model works well but the assumption of eq. (3.23) seems artificial, which can be improved by taking new charge assignment

$$L_1 : 2, \quad L_2 : 1, \quad L_3 : 1, \quad N_1 : 2, \quad N_2 : 1, \quad N_3 : -1. \tag{3.25}$$

In this case the Dirac and Majorana mass matrices become

$$m \simeq \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon \\ \varepsilon^3 & \varepsilon^2 & 1 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix} m_0, \quad M \simeq \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon \\ \varepsilon^3 & \varepsilon^2 & 1 \\ \varepsilon & 1 & \varepsilon^2 \end{pmatrix} M_0, \tag{3.26}$$

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respectively. Each element has an $\mathcal{O}(1)$ coefficient and the ranks of mass matrices are 3. In the diagonal basis of the Majorana mass matrix, eq. (3.26) becomes

$$m_R \simeq \begin{pmatrix} \varepsilon^4 & \varepsilon & \varepsilon \\ \varepsilon^3 & 1 & 1 \\ \varepsilon^3 & 1 & 1 \end{pmatrix} m_0, \quad M_d \simeq \begin{pmatrix} \varepsilon^4 & & \\ & -1+\varepsilon^2 & \\ & & 1+\varepsilon^2 \end{pmatrix} M_0. \tag{3.27}$$

Setting $M_0 \simeq \frac{1}{2R}$, the 5D seesaw induces

$$m_\nu^{5\mathrm{D}} \simeq -\frac{\varepsilon^2 m_0^2}{2\pi R M_* M_0} \left[\begin{pmatrix} \varepsilon \\ 1 \\ 1 \end{pmatrix} (\varepsilon \ 1 \ 1) + \delta' \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} \right], \tag{3.28}$$

where $\delta' = \frac{\pi R M_0}{\varepsilon^2 \tan[\pi R M_0]}$. The 2nd mass matrix has rank 2[8]. The coefficients of M_i are absorbed into those of the mass matrices in eq. (3.28). The SRND structure is obtained when $\delta' \sim 0.1$, where the 1st term in eq. (3.28) is dominant. The reduction of the determinant in the 2-3 sub-matrix is automatic and the NH mass spectrum becomes $\delta'\varepsilon^2 : \delta' : 1$ in units of $\frac{\varepsilon^2 m_0^2}{2\pi R M_* M_0}$. Again, $\mathcal{O}(1)$ coefficients can induce the suitable mixing angles [20].

Finally, let us show one more example of the application of the tanh-function. We take the charge assignment to be

$$L_1 : 1, \quad L_2 : -1, \quad L_3 : -1, \quad N_1 : 1, \quad N_2 : 0, \quad N_3 : 0. \tag{3.29}$$

Then, the Dirac and Majorana mass matrices are given by

$$m \simeq \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ 1 & \varepsilon & \varepsilon \\ 1 & \varepsilon & \varepsilon \end{pmatrix} m_0, \quad M \simeq \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} M_0, \tag{3.30}$$

respectively. The Majorana mass spectrum is $\varepsilon^2 : 1 : 1$ in units of M_0. In the diagonal basis of the Majorana mass matrix, the Dirac mass matrix does not change the order of each element. Then, the light neutrino mass matrix is given by

$$m_\nu^{5\mathrm{D}} \simeq -\frac{m_0^2}{2M_* \tanh[\pi R \varepsilon^2 M_0]} \left[\begin{pmatrix} \varepsilon^2 \\ 1 \\ 1 \end{pmatrix} (\varepsilon^2 \quad 1 \quad 1) + \delta'' \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \end{pmatrix} \right], \tag{3.31}$$

where $\delta'' = \frac{\tanh[\pi R \varepsilon^2 M_0]}{\tanh[\pi R M_0]}$. The 2nd mass matrix has rank 2. The seesaw scales become degenerate due to the tanh-function when $|M_1^S| > \mathcal{O}(R^{-1})$. Both $\tanh[\pi R \varepsilon^2 M_0]$ and $\tanh[\pi R M_0]$ become 1 ($\delta'' \simeq 1$), and hence eq. (3.31) becomes

$$m_\nu^{5\mathrm{D}} \simeq -\frac{m_0^2}{2M_*} \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & 1 & 1 \\ \varepsilon^2 & 1 & 1 \end{pmatrix}. \tag{3.32}$$

Notice that the determinant of the 2-3 sub-matrix is automatically reduced to $\mathcal{O}(\varepsilon^2)$. This mass matrix can induce bi-large mixing with a NH mass spectrum by suitable $\mathcal{O}(1)$ coefficients [20].

[8] If we take $|M_3| = \frac{1}{2R} (= \mathcal{O}((1+\varepsilon^2)M_0))$, the 2nd mass matrix in eq. (3.28) is only produced by $|M_2| = \mathcal{O}((-1+\varepsilon^2)M_0)$, and has rank 1.

4. Summary

In the 5D theory in which the 3 generations of right-handed neutrinos are in the bulk, the neutrino flavor mixings and the mass spectrum can be constructed through the 5D seesaw. We have presented a simple calculation method of the 5D seesaw, in which one just replaces the Majorana mass eigenvalues according to eqs. (2.10) and (2.11). This technique should be useful for studies of neutrino mass textures. The neutrino flavor mixings and the mass spectrum can be drastically changed by the 5D seesaw. The 5D features appear when the Majorana mass is the same as the compactification scale or larger than it. Depending on the type of the bulk mass, the seesaw scales of the 3 generations are strongly split or degenerate. In the split case, the seesaw enhancement is naturally realized. We have shown that the SRND works in a simple setup in 5 dimensions. This enhancement of the effective Majorana mass hierarchy makes the tiny symmetry breaking induce the suitable flavor structure in the $L_\mu - L_\tau$ symmetry. The 5D seesaw can naturally derive some specific mass textures which are just assumptions in the 4D setup. The degenerate case can also realize the suitable flavor structure.

We have only considered the situation that all the 3 generations' seesaw scales are characterized either by the tan- or tanh-function. It should be considered that different generations have different seesaw scales characterized by tan- or tanh-functions (eqs. (2.12) and (2.13)) and induce the neutrino flavor structure. The intermediate scale (such as $10^{9\sim13}$ GeV) can be effectively obtained even by lower scales of M_*, M^V, R^{-1} due to the tan-function. It is also valuable to examine the leptogenesis in this framework. The extension to the warped extra dimension [21] might also induce new possibilities [22].

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